Supplementary Derivations for Interpreting Section 113(2) the Canada Pension Plan Act

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This supplementary note offers additional derivations to clarify discrepancies between Tombe (2023)'s and LifeWorks (2023)'s interpretation of Section 113(2) of the Canada Pension Plan Act. In particular, it presents a detailed mathematical representation of the language in the Act. This representation is, under certain conditions, identical to the Tombe (2023) interpretation and, importantly, offers a precise restatement of the LifeWorks (2023) "alternative interpretation" that differs from this in only one specific respect.

The principle difference between the two interpretations concerns the proportion of each dollar contributed to the national plan that is allocated towards funding benefit expenditures versus saving. That is, should a province-specific savings rate be used or a national savings rate? I show below that investment returns derived from plan contributions depend only on year-specific rates of return and savings rates. Using a uniform savings rate applied to each dollar of contributions, regardless of where that dollar came from, is consistent with the underlying national nature of the CPP, where all contributions are pooled. It is also consistent with historical practice within the CPP. The CPP's original investment income was almost exclusively derived from interest on loans to provincial governments, which were distributed among provinces in proportion to their share of total contributions. This approach is precisely the one implied by each dollar being divided similarly between benefits and savings, irrespective of origin.

This principle is still relevant today, even though the nature of CPP investments has evolved to include a globally diversified fund of largely private securities. Contributions from individual workers have never been earmarked for benefits to any particular retiree or group of retirees but rather have been pooled. Therefore, apportioning investment returns on the basis of contribution shares is more consistent with this historical and ongoing principle. This approach contrasts with the Alberta government's preferred method, which suggests a different allocation for each dollar from Alberta compared to those from other Canadians.

No matter what you think about the right way to read Section 113(2) of the Canada Pension Plan Act, this document is still useful because it creates a new mathematical expression that includes both Tombe (2023) and LifeWorks (2023) as special cases, even though they were both developed separately.

1 Text of the Act

I begin with a restatement of the specific language found in Section 113(2), which details the amount to be paid to a withdrawing province from the CPP fund. It begins with a reference to subsection (1) that

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instructs the federal Minister of Finance to pay an amount to the province as calculated in subsection (2), as follows:

> **Amount to be paid to government of province**
>
> (2) For the purposes of subsection (1), the amount to be calculated as provided in this subsection in the case of any province shall be calculated by the Minister of Finance as the amount obtained by adding
>
> (a) the total amount of all contributions credited to the Canada Pension Plan Account and the Additional Canada Pension Plan Account, to the day on which the regulation referred to in subsection (1) became effective, in respect of employment in that province or in respect of self-employed earnings of persons resident in that province, and
>
> (b) the part of the net investment return of the Investment Board and all interest credited to or accrued to the credit of the Canada Pension Plan Account and the Additional Canada Pension Plan Account, to the day on which the regulation referred to in subsection (1) became effective, that is derived from the contributions referred to in paragraph (a),
>
> and subtracting from the total so obtained
>
> (c) such part of all amounts paid as or on account of benefits under this Act as would not have been payable under this Act if that province had been a province described in paragraph (a) of the definition province providing a comprehensive pension plan in subsection 3(1), and
>
> (d) the part of the costs of administration of this Act, to the day on which the regulation referred to in subsection (1) became effective, that is equal to the proportion of those costs that the total amount of the contributions referred to in paragraph (a) is of the total amount of all contributions credited to the Canada Pension Plan Account and the Additional Canada Pension Plan Account to that day.

As Tombe (2023) emphasizes, paragraph (b) is particularly problematic. The "part of the net investment return of the Investment Board ... credited to or accrued to the credit of the Canada Pension Plan Account ... that is derived from the contributions" could be interpreted in different ways and hinges on what "derived from" means in this context. There are also important limitations in the publicly available data that prevent a precise implementation of this approach, even if the language in the Act were clear. I will not restate the arguments put forward in that paper here. I turn instead to a precise mathematical representation of CPP asset accumulation to make clear how investment returns are derived from contributions and under what conditions particular interpretations apply.

2 The Algebra of Pension Plan Asset Accumulation

Consider a pension plan with accumulated investment assets A_t. These assets evolve according to the difference between contributions C_t and benefit expenditures B_t, plus any investment income derived from the last period's assets $r_t \times A_{t-1}$. If all payments come at the end of the period, then

$$A_t = A_{t-1}(1 + r_t) + C_t - B_t, \tag{1}$$

Iterating over time, starting from period $t = 0$ where there are no assets and therefore $A_0 = 0$, we have assets at some end point T given by

$$A_T = \varphi_T \left(\sum_{t=1}^{T} \varphi_t^{-1} (C_t - B_t) \right), \tag{2}$$

where

$$\varphi_t = \prod_{i=1}^{t} (1 + r_i) \tag{3}$$

captures interest rates r_t that vary over time and accumulate returns from period 1 to t. Intuitively, the value of assets at time T will equal the present value of all future annual plan surpluses $(C_t - B_t)$ projected forward to time T using φ_T.

2.1 Investment Returns

The value of assets reflect contributions of principle (from surpluses) each year $(C_t - B_t)$ and income derived from those surpluses that compound over time. Those returns are equal to

$$R_T = \sum_{t=1}^{T} \left(\frac{\varphi_T}{\varphi_t} - 1 \right) (C_t - B_t) \tag{4}$$

For additional intuition behind this equation, consider that period 1 savings generates $r_2 \times (C_1 - B_1)$ returns in period 2. The period 1 savings generates $r_3 \times (1 + r_2) \times (C_1 - B_1)$ returns in period 3. And so on. Total returns in both periods 2 and 3 from savings in period 1 are $(r_3(1 + r_2) + r_2)(C_1 - B_1)$, or simply $(1 + r_3)(1 + r_2)(C_1 - B_1) - (C_1 - B_1)$. Notice this is $A_3 - (C_1 - B_1)$. This can be similarly applied to any period's surplus across any time period, which yields total investment returns generated from period 1 to T must be $A_T - \sum_{t=1}^{T} (C_t - B_t)$. The above expression for returns can therefore be equivalently expressed as:

$$R_T = \sum_{t=1}^{T} \left(\frac{\varphi_T}{\varphi_t} - 1 \right) C_t \left(1 - B_t / C_t\right), \tag{5}$$

which is more relevant for interpreting the language of the Canada Pension Plan Act. Take note that accumulated income by period T reflects investment returns over time (captured by $\varphi_T/\varphi_t - 1$) and the proportion of each dollar in contributions that goes toward savings (captured by $1 - B_t/C_t$). Given that savings rate, the returns derived from the particular subset of contributions from province i, denoted R_{iT}, would simply be

$$R_{iT} = \sum_{t=1}^{T} \left(\frac{\varphi_T}{\varphi_t} - 1 \right) C_{it} \left(1 - B_t / C_t\right), \tag{6}$$

which aggregates since $\sum_i R_{iT} = R_T$.

2.2 Mathematical Expression for Section 113(2)

Paragraph (b) of Section 113(2) refers to the "part of the net investment return ... that is derived from the contributions." Equation 5 maps directly into that language since returns are derived from contributions according to (1) investment returns (captured by φ_T/φ_t) and savings rates (captured by $1 - B_t/C_t$). With

this, the rest of Section 113(2) follows in a straightforward manner.

The amount paid to a withdrawing province would be the sum of

- Total contributions paid into the CPP from that province: $\sum_{t=1}^{T} C_{it}$,
- Total income derived from those contributions: $\sum_{t=1}^{T} \left(\frac{\varphi_T}{\varphi_t} - 1\right) C_{it} \left(1 - B_t/C_t\right)$

less

- Total expenditures paid out that are attributable to the province: $\sum_{t=1}^{T} B_{it}$
- Total costs of administration attributable to the province $\left(\frac{\sum_{t=1}^{T} C_{it}}{\sum_{t=1}^{T} C_t}\right) \times \sum_{t=1}^{T} E_t \equiv c_i \sum_{t=1}^{T} E_t$

where E_t are the costs of administration referred to in paragraph (d). The language in paragraph (d) is unambiguous and requires allocating these costs to a withdrawing province on the basis of its total gross contribution share, denoted c_i.

All together, Section 113(2) would, under this interpretation, pay out an amount A_{it} to province i at withdrawal date T,

$$A_{iT} = \left[\sum_{t=1}^{T} C_{it}\right] + \left[\sum_{t=1}^{T} \left(\frac{\varphi_T}{\varphi_t} - 1\right) C_{it} \left(1 - B_t/C_t\right)\right] - \left[\sum_{t=1}^{T} B_{it}\right] - \left[c_i \sum_{t=1}^{T} E_t\right]. \tag{7}$$

This provides for a division of assets that perfectly aggregates since $\sum_i A_{iT} = A_T$, although, as we will see in the next section, so too does the LifeWorks approach.

Importantly, notice also that the share of each dollar in contributions from employment in province i that is saved is $(1 - B_t/C_t)$, which is independent of which province is withdrawing from the plan. A common year-specific savings rate applied to any withdrawing province follows from CPP contributions being pooled into a single national fund. There are no earmarks of contributions to fund benefits for a specific retiree or any subset of retirees. As a purely normative matter, one may regret participating in a national plan for any number of reasons. But that doesn't change the fact that all included provinces have been participating in a national plan.

The above also contains the more compact representation from Tombe (2023) in the special case where contribution shares C_{it}/C_t are stable at c_i for all time. In that case, we have the following paid to a withdrawing province:

$$\sum_{t=1}^{T} (C_{it} - B_{it}) + c_i \times R_T. \tag{8}$$

This follows because $C_{it}(1 - B_t/C_t) = (C_{it}/C_t)(B_t - C_t)$ and therefore if $(C_{it}/C_t) = c_i$ then

$$\begin{aligned} \sum_{t=1}^{T} \left(\frac{\varphi_T}{\varphi_t} - 1\right) C_{it} \left(1 - B_t/C_t\right) &= \sum_{t=1}^{T} \left(\frac{\varphi_T}{\varphi_t} - 1\right) \frac{C_{it}}{C_t} \left(C_t - B_t\right), \\ &= c_i \times \left[\sum_{t=1}^{T} \left(\frac{\varphi_T}{\varphi_t} - 1\right) \left(C_t - B_t\right)\right], \\ &= c_i \times R_T, \end{aligned} \tag{9}$$

where the last line follows from equation 4. Of course, contribution shares are not stable over time, but

the variation over time in contribution shares does not result in overly large quantitative implications for amounts paid, in Alberta's case, relative to an alternative interpretation of the Act that I describe next.

2.3 The LifeWorks Interpretation

The LifeWorks approach takes the year-by-year surplus of contributions from employment in Alberta C_{it} over expenditures to Alberta recipients B_{it} and accumulates this over time at the same annual rate of return as CPP investments r_t. Ignoring costs of administration, this results in

$$A_{it} = A_{i,t-1}(1 + r_t) + C_{it} - B_{it}, \tag{10}$$

and therefore

$$A_{iT} = \varphi_T \left(\sum_{t=1}^{T} \varphi_t^{-1} (C_{it} - B_{it}) \right). \tag{11}$$

By the end of 2021, this approach yields approximately $280 billion. As in the previous section, one can separate the contribution of accumulated principle from returns, which yields

$$A_{iT} = \sum_{t=1}^{T} (C_{it} - B_{it}) + \sum_{t=1}^{T} \left(\frac{\varphi_T}{\varphi_t} - 1 \right) (C_{it} - B_{it}). \tag{12}$$

Isolating the annual contributions C_{it} within the summation, we have returns derived from contributions under this approach of

$$R_{iT} = \sum_{t=1}^{T} \left(\frac{\varphi_T}{\varphi_t} - 1 \right) C_{it} \left(1 - B_{it}/C_{it}\right). \tag{13}$$

Although the savings rate applied to contributions is province- and year-specific, rather than just year-specific as in the previous section, the LifeWorks approach does appropriately aggregate since

$$\begin{aligned}
\sum_i A_{iT} &= \sum_i \sum_{t=1}^{T} (C_{it} - B_{it}) + \sum_i \sum_{t=1}^{T} \left(\frac{\varphi_T}{\varphi_t} - 1 \right) (C_{it} - B_{it}), \\
&= \sum_{t=1}^{T} (C_t - B_t) + \sum_{t=1}^{T} \left(\frac{\varphi_T}{\varphi_t} - 1 \right) \sum_i (C_{it} - B_{it}), \\
&= \sum_{t=1}^{T} (C_t - B_t) + \sum_{t=1}^{T} \left(\frac{\varphi_T}{\varphi_t} - 1 \right) (C_t - B_t), \\
&= A_T.
\end{aligned} \tag{14}$$

The only difference between the two approaches (that is, between equations 6 and 13) is therefore just the implied fraction of each dollar contributed to the CPP that is saved versus used to fund current expenditures. The LifeWorks approach assumes each dollar contributed by an Alberta employee and employer is split according to $(1 - B_{it}/C_{it})$ rather than $(1 - B_t/C_t)$. These are very different values. I estimate that between 1966 and 2021, the average share of contributions in the entire CPP used to fund benefits was approximately 95 percent. And therefore, only 5 percent are saved. In Alberta, meanwhile, the shares are approximately 64 and 36 percent, respectively.

3 Additional Discussion

What is the appropriate savings rate to apply to contributions, $(1 - B_{it}/C_{it})$ or $(1 - B_t/C_t)$? Are there alternative rates that may be admissible under a reasonable interpretation of Section 113(2)?

The rule implied by equation 7 implies year-specific gross contribution shares should be used to apportion national excess contributions. Equivalently, contributions from any province should be split identically between saving and benefits but should also be year-specific. This implies that the share of total accumulated national investment returns derived from contributions R_T that are paid to a withdrawing province i, denoted R_{iT}, is some weighted average of gross contribution shares over time. Specifically,

$$R_{it} = \sum_{t=1}^{T} \left(\frac{\varphi_T}{\varphi_t} - 1 \right) \frac{C_{it}}{C_t} \left(C_t - B_t \right), \tag{15}$$

$$= \sum_{t=1}^{T} \frac{C_{it}}{C_t} \frac{\left(\frac{\varphi_T}{\varphi_t} - 1 \right) \left(C_t - B_t \right)}{\sum_t \left(\frac{\varphi_T}{\varphi_t} - 1 \right) \left(C_t - B_t \right)} R_T, \tag{16}$$

$$\equiv \left(\sum_{t=1}^{T} \frac{C_{it}}{C_t} \omega_{it} \right) \times R_T, \tag{17}$$

Intuitively, the weight given to any particular year's gross contribution share from province i depends on the "importance" of that year for accumulating investment income within the CPP, either because of a large national excess $(C_t - B_t)$ or because of a large return between that year and the withdrawal period (φ_T/φ_t). If contribution shares are stable over time, then $\left(\sum_{t=1}^{T} \frac{C_{it}}{C_t} \omega_{it} \right) = c_i$.

While year-by-year apportioning of contributions is most elegant, there are alternatives that have historical grounding. A 10-year rolling average of contribution shares, for example, was used historically under the original apportionment of national excess contributions when the CPP only purchased provincial bonds. Using the entire history of contributions as a single aggregate share, that is, c_i, is used to apportion CPP operating costs under Section 113(2)(d). These alternatives do matter, but only modestly. For the period 1966–2021, I estimate the year-by-year approach would yield $107 billion to Alberta as of the end of 2021, while a 10-year moving average would yield $100 billion and a single aggregate share would yield $130 billion. While these are meaningful differences, using Alberta-specific savings per dollar (the LifeWorks approach) yields a massively different result. I estimate that for the same 1966–2021 period, $289 billion would be provided to Alberta.

In short, if all contributions are treated identically (that is, split similarly between saving and funding benefits) regardless of the origin of those contributions, then a range of 20 to 25 percent is potentially appropriate for A_{iT}. To arrive at a value for A_{iT} that is more than half the CPP, as in the LifeWorks approach, one must assume contributions from a specific province are earmarked to fund a specific subset of beneficiaries, which is at odds with the nature of the CPP as a national plan. Whatever the ultimate formula that is applied (if ever), this brief note hopefully helps clarify some of the underlying mathematical differences between two leading approaches within the APP policy debate.

References

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